Strategic R&D policies with spillovers and trade liberalisation

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Abstract

We examine the impact of trade liberalisation and R&D policies on exporting firms' incentive to innovate and social welfare. Key factors determining the government's optimal policy are the strength of R&D spillover effect and the toughness of firm competition. When domestic exporting firms only compete in an overseas market, the optimal policy might be to tax firms' R&D. Trade liberalisation in the overseas market induces a higher R&D tax rate. When the firms also conduct business in home market, the government should financially support firms' R&D. Trade liberalisation always increases firms' output sales, R&D investments, and social welfare. In an international context where there is competition between exporting firms located in different countries, while forming an international R&D joint venture ensures a symmetric outcome, further international cooperation may lead to an asymmetric equilibrium with only one firm being subsidised on its R&D investment.
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1 Introduction

Over the last few decades, there has been an increasing number of countries that adopt export promoting trade strategy for their economic development path. Following this strategy, firms are encouraged by their national governments to export their products to an overseas market. Expected gains of this trade policy are many, of which the most visible ones include generating foreign exchange revenue, increasing employment and improving production efficiency. However, the successful implementation of this policy is not always compelling. On the one hand, it depends on external factors such as demand and regulations in the international market. On the other hand, it is subject to the competitiveness of exporting firms. Hence, in order to survive and develop in such a competitive market place, firms need to improve their productivity and in that process, innovation is essential. From a policy standpoint, a government can support their domestic exporting firms by providing them with either export or R&D subsidies. However, as export subsidies are often restricted due to international agreements, providing subsidies to firms' R&D activities become the most effective policy tool of any national governments nowadays. Several studies such as Spencer and Brander (1983), Bagwell and Staiger (1994), Brander (1995), Neary and Leahy (2000), and Leahy and Neary (2001) even find that subsidising R&D is more powerful than subsidising exports.

Clearly, trade liberalisation and R&D policies are closely related. While trade liberalisation affects factors impacting innovation activities such as market size and toughness of competition, R&D investment determines the benefits of undertaking the trade. It is surprising that not much has been done to examine the links between these two policy factors although there exists rich branches of literature studying each factor separately. Filling this gap will be the main task of this paper. In doing so, this paper considers the issue of exporting duopoly in a basic model of strategic R&D with trade liberalisation occurring in exporting market(s).[1] Here, firms produce horizontally differentiated products and invest in R&D to reduce their marginal cost of production. R&D investment has an important feature that it benefits both its own investor and other firms (through an R&D spillover process). Government policies include providing an R&D subsidy to the exporting firms. This environment creates a two-stage game which can be solved by backward induction. In the first stage, the government decides on how much to subsidise R&D activity of firms in order to maximise domestic welfare.[2] In the second stage, firms maximise their profits by choosing levels of R&D investment then export volumes and/or domestic sales optimally taking into account the subsidy rate provided by the government and the other firm's action. The result at the end of the second stage is a Cournot-Nash equilibrium. Depending on the setting environment, the strategic behaviours of the government(s) and firms are different and convey different implications for the optimal R&D subsidy.

[1]Although it is arguable that exports and imports are highly connected, for the purpose of focusing on public policies towards supporting exporting firms, this paper does not consider imports.

[2]In this paper, it is assumed that the government has full credibility towards its policy. This means that it can always and is willing to commit to its policy instrument. Having said so, we fully acknowledge an expanding literature that studies the issue of timing and non-commitment to actions of the governments such as Baghdasaryan and Zigic (2010) and Zigic (2011) among others. Zigic (2011) provides an excellent review of that literature.

The first results are developed in a simple setting with two domestic exporting firms competing in an overseas niche market. For simplicity, foreign firms are assumed either non-existent or too small to count on (i.e. they hold a negligible market share or operate in a completely different market segment).[3] The result indicates that factors that shape the government's policy action are the strength of R&D spillover effect (a social benefit) and the degree of firm rivalry (a social cost). This result is new since existing studies in this branch of literature on trade liberalisation are largely silent about R&D spillover effect. The government's optimal policy might be to tax firms' R&D activity instead of subsidising it. This optimal R&D tax increases when trade liberalisation in the foreign market occurs. This trade liberalisation is also found to induce a higher level of R&D investments of firms, their productivity and export sales, and social welfare.

Results on optimal R&D policy are substantially different when domestic exporting firms also conduct business at home. The first-best policy is always to subsidise R&D of firms. Trade liberalisation implemented by the foreign market does not always induce a higher optimal R&D subsidy level because the extra gain from undertaking further R&D may be smaller than its additional cost. This benefit and cost analysis will pin down the direction of change of the optimal policy tool in the presence of a lower trade cost.

In extending the modelling framework to an international setting that covers competition between firms located in different countries, this paper finds that creating an international R&D joint venture is possible and leads to a symmetric outcome where firms share their R&D information and the governments use R&D subsidies as policy measures to maximise aggregate welfare. Interestingly, when countries are highly integrated, it may be optimal from the whole society's point of view to allow only one firm to operate (i.e. conduct R&D investment and produce output) especially when the fixed cost of R&D investment is sufficiently high (but not too high).

In characterising R&D subsidies, the majority of existing studies (e.g. Brander, 1995; Neary and Leahy, 2000; Leahy and Neary, 2001) only focus on business-stealing motive and pay little attention to the welfare motive of R&D subsidisation. This is because they do not consider any welfare analysis. Collie (2002) is among a few exceptions looking at welfare effect of subsidies but it addresses production subsidies rather than R&D subsidies. Spencer and Brander (1983) and Haaland and Kind (2008) are studies most closely related to our paper in terms of studying R&D subsidisation. However, they only restrict their attention to competition between a home firm and a foreign firm rather than that between two exporting firms as presented in our paper. Long et al. (2011), while studying the impact of trade liberalisation on R&D, do not consider the subsidisation issue. Similar to Neary and O'Sullivan (1999) and Leahy and Neary (2005), that paper

[3] This simplifying assumption allows us to focus better on the strategic behaviour of the exporting firms originated from a same country. The presence of foreign exporting firms will later be considered in the paper.

looks at R&D cooperation/competition between firms rather than R&D coordination by the government at the policy stage. To some extent, our paper is also related to Long and Stahler (2009) in terms of considering strategic behaviour of firms under different scenarios. Nevertheless, their paper focuses on firm ownership and trade policy, not R&D investment and trade policy as our paper does. Our paper also considers the impact of R&D investment externality, an issue that has not been fully explored in the R&D-trade related literature. For example, unlike our paper, in Zigic (2011), R&D spillovers are an unilateral process that takes place from a domestic firm to a foreign firm.[4]

The paper proceeds as follows. In Section 2, we introduce a basic model of competition between exporting firms in an overseas market. In Section 3, we additionally allow exporting firms to trade in their home market. For each case, the existence as well as uniqueness of an optimal R&D subsidy and its key characteristics is analysed. The impacts of trade liberalisation on firms' output sales, their cost-reducing R&D investments and productivity, and social welfare are also examined. Section 4 extends the modelling framework to international competition between firms of different nationalities. It investigates the possibility of different degrees of R&D coordination between the governments and their implications. Section 5 ends the paper with some concluding remarks.

2 The baseline model

Consider two domestic firms i and j whose products are entirely exported to a foreign country that does not produce these goods.[5] This modelling assumption fits well with the case of firms operating in export processing zones (EPZs) where all of the firm's products are to be sold in a foreign market. Sargent and Matthew (2009), in citing statistics provided by The International Labour Organisation, indicate that by 2002, there had been 116 countries establishing EPZs to promote their exports. China is often considered as a successful country in this policy direction.[6] In this setting, the utility function of an overseas representative consumer is:

$$u = \alpha q_i + \alpha q_j - \left(\frac{q_i^2}{2} + \frac{q_j^2}{2} + b q_i q_j\right), b \in [0, 1], \alpha > 0, \tag{1}$$

where q_i and q_j are consumption of the goods produced by the two firms respectively; b denotes the degree of substitution between the two goods (the higher the value of b, the higher the degree of substitutability). When $b = 0$, the goods are completely independent and when $b = 1$, the goods are identical. This quadratic utility function is standard and

[4] This paper also recognises a rich literature evaluating the effects of spillovers on R&D investment equilibrium such as D'Aspremont and Jacquemin (1988), De Bondt and Veugelers (1991) and Kamien et al (1992). However, these studies do not consider international trade or trade liberalisation.

[5] In this setting, the exporting country is referred to as the home country.

[6] For a more detailed review of EPZs around the world and the China's EPZ success, see Sargent and Matthews (2009).

has been used by Haaland and Kind (2008). For simplicity, assume the population size in the foreign market is equal to 1.

Let p_i and p_j denote the prices of the two goods in the foreign country. The consumer surplus of the foreign country can be expressed as $CS = u - p_i q_i - p_j q_j$. As the consumer maximises his surplus with respect to the quantity of each good, the (inverse) demand function for good i (and similar for good j) can be derived as $p_i = \alpha - (q_i + bq_j)$.

Assume that the firms' products are subject to a trade cost (e.g. transportation or service cost) of rate τ per unit of goods they export to the foreign market ($\tau > 0$). By *trade liberalisation* in the overseas market, it is meant an exogenous *fall* in τ. In the absence of R&D, firms face a same unit cost of production, c. These imply that in order to sell their products in the foreign market, firms have to bear the exporting cost of $c+\tau$. To allow firms to be able to export even when no R&D activity is conducted, assume that $\alpha > c + \tau$.[7]

Firms invest in R&D to reduce their cost of production so that the cost of production after R&D is $c - x_k$ where x_k ($c \geq x_k \geq 0$, $k = i, j$) is the amount of R&D effort expended by firms. Define $\lambda \in [0, 1]$ as the degree of R&D spillovers between firms (when $\lambda = 0$, there are no spillovers and when $\lambda = 1$, there are perfect spillovers). Hence, the cost of production of firm i after its R&D investment and the spillovers from firm j will be $c - x_i - \lambda x_j$. The R&D cost function $r(x_k)$ takes a quadratic form as follows:

$$r(x_k) = Mx_k^2 + f, \tag{2}$$

where $f \geq 0$ is the fixed cost for setting up an R&D project and M is a constant satisfying Assumption 1 below. This R&D cost function is initiated by D'Aspremont and Jacquemin (1988) and used extensively by many subsequent papers such as Haaland and Kind (2008). According to Amir (2000), this setting is suitable for certain industries or R&D processes such as technology parks. In that respect, the benefits of R&D spillovers outweigh the negative effects resulting from increased firm competition when firms join these technology parks.[8]

Assumption 1 *Parameters are such that*

$$M > \max\left[\frac{(b+5)(\lambda+1)^2}{2(b+2)^2}; \frac{(\lambda+1)^2\left[(b+5)\alpha - 2\tau\right]}{2(b+2)^2 c}\right].$$

As will be shown later in the Appendix, this assumption is needed for fulfilling sufficient conditions of maximisation problems. Under this assumption, the R&D cost function is,

[7] Look differently, the threshold on trade cost above which trade cannot take place is $\tau = \alpha - c$. This is because when trade cost is higher than this threshold, product prices will be negative.

[8] For further discussion on esoteric aspects of D'Aspremont and Jacquemin's (1988) model, especially as compared to that of Kamien et al. (1992), see Amir (2000).

thus, positively valued, strictly increasing and strictly convex in the level of R&D effort conducted by firms.

Also assume that the government regulates R&D investment by providing an R&D subsidy (tax) of rate s_k ($k = i,\ j$) per unit of R&D investment conducted by each firm.[9] Hence, the profit function for firm i (and similar for firm j) is:

$$\pi_i = [p_i - (c - x_i - \lambda x_j) - \tau]\, q_i - r(x_i) + s_i x_i. \tag{3}$$

Each firm will maximise its profit while the domestic government will maximise total welfare. Because all goods are exported and not consumed in domestic market, domestic consumer surplus is zero. Hence, total welfare is equal to total firms' profits less R&D subsidy costs:

$$W = \sum_{k=i,j} \pi_k - \sum_{k=i,j} s_k x_k. \tag{4}$$

In this paper, we follow Long et al. (2011) in using Melitz (2003)'s definition of productivity. Here, firm i's productivity (and similar for firm j), z_i, is the inverse of its marginal production cost:

$$z_i = \frac{1}{c - x_i - \lambda x_j}, \tag{5}$$

and the industry productivity, Z, is the inverse of the average marginal production cost of that industry:

$$Z = \frac{2}{(c - x_i - \lambda x_j) + (c - x_j - \lambda x_i)}. \tag{6}$$

The above setting provides us with a two-stage game. In the first stage, the government chooses how much to subsidise firms' R&D efforts to maximise social welfare. In the second stage, the firms choose R&D investment levels and export volumes to maximise their corresponding profits taking into account the R&D subsidy rates given in the first stage. We will solve this game using backward induction.

Lemma 1 *Consider a symmetric equilibrium where $s_i = s_j = s$, $q_i = q_j = q$, $x_i = x_j = x$. If it is interior,*[10] *then*

$$s = \frac{(2\lambda - b)q}{b + 2}. \tag{7}$$

Proof. See Appendix.

This result indicates that in this symmetric equilibrium, firms receive the same amount of subsidy from the government, undertake the same amount of R&D investment, and exports the same quantity of goods to the foreign market.[11] It also shows the relationship

[9] Note that when $s < 0$, it is an R&D tax instead.

[10] Only in Proposition 1 below, under some further assumptions, the solution is actually interior.

[11] Clearly, the incentive for the government to subsidise its home firms competing in a foreign market comes from welfare maximisation. Another reason for such a policy tool, although not considered in this paper, is to enhance employment. One can also think of this policy action as a result of lobbying. However, we abstract from this aspect for simplicity.

between the R&D subsidy and the export volume. Although the government subsidises the firms' R&D investment, their implemented policy indirectly affects the quantity of goods that firms want to sell in the foreign market.

This important result links the externality of R&D investment, λ, with the degree of substitutability between the goods, b. In particular, if $\lambda = \frac{b}{2}$, $s = 0$; when $\lambda < \frac{b}{2}$, $s < 0$ and when $\lambda > \frac{b}{2}$, $s > 0$. From the whole society's point of view, λ represents the social benefit of undertaking R&D because R&D is not only good for its own investor but also others in the market. By contrast, b is somewhat a social cost to the exporting country as it reflects the rivalry between the exporting firms in the overseas market.[12] When $\lambda = \frac{b}{2}$, the social benefit of undertaking R&D investment cancels out with the corresponding social cost so the government has no incentive to finance firms' R&D activity.

Note that a special case that satisfies the condition $\lambda = \frac{b}{2}$ happens when $\lambda = 0$ and $b = 0$. In this case, there will be no R&D spillovers between firms and goods are absolutely different (i.e. firms are independent monopolies in their product lines and facing no competition from each other). The normal wisdom is that due to absence of competition ($b = 0$), there is no need for the government to help the firms further exploit their monopoly power in the overseas market. That is true but not enough. An additional condition is that there is no R&D spillovers between the firms. Even when firms are monopolies but if there are R&D investment spillovers, the social benefit of undertaking R&D is high (firms benefit from each other's R&D investment implementation), the government has an incentive to support the firms because this action is welfare enhancing. However, if there is no R&D investment externality, the government is willing to leave the firms untouched. In this case, each firm's marginal export revenue and its marginal R&D spending cost cancel out each other. Any firm's extra profit will be equal to the value of R&D subsidy it receives from the government. Consequently, the government cannot use R&D subsidy to increase the exporting firms' profit net of R&D subsidy cost for the welfare. This indicates that the optimal policy for the government is to withhold any R&D subsidy to the firms.

Given that under some circumstances, the government may need to tax R&D investment (i.e. setting $s < 0$ when $\lambda < \frac{b}{2}$). This tax is required to reduce firms' excessive R&D spending. This leads to the question about having only one firm engaging in R&D activity instead. Upon investigating this possibility, we can state the following:

Lemma 2 *Suppose that the government is able to differentiate between the firms and wants only one firm to undertake cost reducing innovation. The government then considers to tax R&D investment of one firm while still subsidising the other. However, such*

[12] If the two home firm fail to jointly internalise the national monopoly power on the foreign market, this degree of rivalry will reduce total profit which, in turn, will reduce the home country's welfare. According to Haaland and Kind (2008), an increase in b implies a decrease in market demand. In other words, the size of the market gets smaller when goods become less differentiated.

an asymmetric policy is not optimal.

Proof. See Appendix.

The results indicate that there may be an asymmetric policy that leads to having only one firm conducting research in the market. As shown in the Appendix, under such a policy, the favourable firm does not receive any financial support from the government as any subsidy provided will be welfare reducing. In the meantime, the unfavourable firm is penalised for any research effort made. Both firms are shown to produce positive outputs.

It should be noted that although such an asymmetric policy exists, it is not optimal. The corresponding welfare is smaller than that obtained under the symmetric case. In addition, to put forward this asymmetric arrangement, the government will have to pursue a discriminatory policy because firms are treated differently. This policy may not be easily conducted in practice due to restrictions set out in national laws. As a result, from now on we will only focus on equilibria arising from a non-discriminatory (symmetric) setting.

From the lemmas above, we obtain results that can be summarised in the proposition below:

Proposition 1 *Consider a symmetric equilibrium. When exporting firms only compete in an overseas market and* $\lambda \neq \frac{b}{2}$*, the social optimum can be achieved as an interior Nash equilibrium with the government taking action towards firms' R&D activities. In particular, if* $\lambda > \frac{b}{2}$*, it is optimal to subsidise firms' R&D investment; otherwise, an optimal R&D tax is required. Trade liberalisation in the foreign market induces a higher level of optimal R&D subsidy provided (optimal R&D tax imposed) if there has been such a subsidy (tax) in place.*

Proof. See Appendix.

This proposition contains two important results. The first result says that the socially optimal policy may be that the government taxes firms' R&D activity instead of subsidising them. This can be explained on the following ground. When firms conduct R&D and then compete with each other in a foreign market, there are two important factors affecting welfare of the entire economy. While the R&D spillovers effect (captured by λ), a positive externality, enhances domestic welfare, the rivalry of firms (reflected through b), a negative externality, reduces it. In particular, when the R&D spillover intensity is relatively small as compared to the degree of competition between firms ($\lambda < \frac{b}{2}$), the competition of firms result in a net effect in which the home country as a whole fails to fully exploit its potential monopoly power in that foreign market. Too much R&D conducted will lead to the situation of over-production for the two domestic exporting firms. To avoid this situation, the home government should impose an R&D tax, at the same rate, on both firms. This optimal R&D tax guarantees that social welfare will be maximised and firms will have no incentive to do less or more R&D and, hence, to produce less or more exported products. By contrast, when $\lambda > \frac{b}{2}$, the benefit of increasing R&D is greater than its cost, providing an R&D subsidy is the optimal policy action that the government should pursue.

The second result says that when there is a reduction in the trade cost, the optimal action of the home government is to tax the firms' R&D investments more heavily if there is already a tax or to provide the firms with more financial support if there is already a subsidy in place. This is because lower trade cost expands firms' export volumes and thus raises firms' willingness to invest in cost-reducing R&D. If the social benefits of conducting more R&D is larger than its associated social costs (through fiercer firms' competition), a reduction in the trade cost induces a higher level of optimal R&D subsidy. However, in case that an R&D tax is needed, to reduce firms' excessive R&D spending so that overproduction, which erodes the home country's monopoly power in the foreign market, can be avoided, the government needs to raise the R&D tax rate. This action will result in an improvement in social welfare because (i) when there is an R&D tax and a higher tax rate is imposed, firms obtain more profits from exports (even though no more R&D investments occur) and the government collects more R&D tax revenues; and (ii) when there is an R&D subsidy, the extra profits obtained by the firms exceed the R&D subsidy costs expended by the government.

We now examine the economic impact of trade liberalisation on the home country. To derive the comparative static effects of a reduction in τ, we differentiate the obtained equilibrium conditions with respect to τ. The results can be summarised in the proposition below:

Proposition 2 *When exporting firms only compete in a foreign market, at the optimal policy action conducted by the government, trade liberalisation in the foreign market raises firms' cost-reducing R&D spending, their productivity and the industry productivity. It also enhances domestic welfare.*

Proof. See Appendix.

The results obtained can be explained as follows. Basically, trade liberalisation in the foreign market entails two different effects: a *direct effect* and an *indirect effect*. The direct effect of a fall in the trade cost, as explained under Proposition 1, encourages firms to conduct more cost-reducing R&D. By contrast, the indirect effect influences firms' R&D efforts through changing the optimal R&D policy instrument. In case of an optimal R&D subsidy, the two effects complement each other. However, in case of an optimal R&D tax, although the two effects work in opposite directions, the direct effect dominates the indirect one resulting in a net positive effect of an increase in R&D investments for the firms. Hence, there will be an improvement in firms' and industry's productivity as well as export volumes (because the whole exporting cost is lower). This sale expansion allows firms to enjoy higher profits and the increment in profits is more than required to offset for the increase in government's subsidy expenditure. In the case of a tax, the government gets more revenue through its higher R&D taxation program. All this leads to a higher level of domestic welfare.

3 Adding domestic sales

In the previous section, firms are assumed to sell all of their products overseas. Because there is no domestic consumption of firms' product, only firms' export sales and government expenditure/revenue matter for the social welfare. If this assumption is relaxed, i.e. if firms are allowed to sell their products in the home market, the strategic behaviours of firms and the home government are expected to change significantly. This is because firms will now weigh up between selling products at home and overseas. In addition, the government will now need to take into account consumer surplus in calculating the social welfare. To examine this interesting case, we slightly restructure our model below.

In addition to the competition in the foreign market as described in the previous section, we now further assume that competition between two exporting firms also takes place in the home market.[13] As there are now two markets, we need to make some small changes in notation. Define the home market as Country h and the foreign market as Country f. Assume the population size in each country is equal to 1 and consumers everywhere have the same preferences for simplicity. The representative consumer in home country derives utility from consuming goods supplied by the firms:

$$u_h = \alpha q_{ih} + \alpha q_{jh} - \left(\frac{q_{ih}^2}{2} + \frac{q_{jh}^2}{2} + b q_{ih} q_{jh} \right), \, b \in [0, 1), \alpha > 0, \tag{8}$$

and similar for the consumer in the foreign country. Here, q_{ih} and q_{jh} denote the domestic consumption of goods produced by the firms. The first subscript is used to indicate the firm producing the consumption good and the second subscript refers to the country of consumption. The domestic consumer surplus is $CS_h = u_h - p_{ih}q_{ih} - p_{jh}q_{jh}$. From this, the inverse demand function for firm i's product (and similar for firm j's product) is $p_{ih} = \alpha - (q_{ih} + bq_{jh})$. Using these results, the maximised domestic consumer surplus can be calculated as $CS_h = \frac{1}{2}\left(q_{ih}^2 + q_{jh}^2\right) + bq_{ih}q_{jh}$. The inverse demand functions for goods in the overseas market are the same as described previously. Hence, the profit function for firm i is:

$$\pi_i = [p_{ih} - (c - x_i - \lambda x_j)]\, q_{ih} + [p_{if} - (c - x_i - \lambda x_j) - \tau]\, q_{if} - r(x_i) + s_i x_i, \tag{9}$$

and similar for firm j. In this profit function, the first two terms capture the firm's domestic sales revenue and export sales revenue respectively while the last two terms are

[13] The setting in this section not only accommodates well the recent move of several countries, e.g. China or Vietnam, that additionally allow EPZ firms to sell a certain fraction of their products to their corresponding domestic markets.

R&D investment spending and financial support from the government.

Welfare of the home country will be:

$$W_h = \pi_i + \pi_j + CS_h - s_i x_i - s_j x_j. \tag{10}$$

A slight difference between this welfare function and the one defined in the previous section is the inclusion of consumer surplus. Any R&D policies should now also take this component into account.

Lemma 3 *Assuming interior solution then there exists a symmetric equilibrium outcome such that $s_i = s_j = s$, $q_{ih} = q_{jh} = q_h$, $q_{if} = q_{jf} = q_f$, and $x_i = x_j = x$.*

Proof. See Appendix.

This lemma provides us with interior equilibrium levels of domestic sale, export sale and R&D investment of the firms. It also spells out the condition on the equilibrium R&D subsidy following which a unique optimal solution to the firms' maximisation problem is obtained. Given this setting and conditions, we can derive the following:

Proposition 3 *When firms compete in both home and foreign markets, the welfare maximising R&D subsidy expended by the government to each firm exists, is positively valued and uniquely determined. In addition, trade liberalisation in the foreign market will induce an increase in this optimal R&D subsidy level only if (i) $\lambda \geq \frac{b}{2}$; or (ii) $\lambda < \frac{b}{2}$ and M is moderately high.*

Proof. See Appendix.

The obtained results deserve some comments. Unlike the results obtained under Proposition 1 where an R&D tax might be imposed, when firms also trade in the home market, the government's optimal policy is always to subsidise R&D. This is very much because of the consumer surplus motive. In this case, the gain in consumer surplus due to R&D subsidy, which lowers the product prices by lowering firms' marginal production cost, is more than sufficient to compensate for the associated costs (incurred through R&D subsidy expenditure) so the government has an incentive to grant R&D subsidy to the firms.

Another difference is that the effect of trade liberalisation in the foreign market on optimal R&D subsidy, to some extent, is also dependent on cost structure of the R&D investment (that is captured by the magnitude of M).[14] When the intensity of R&D spillovers is relatively large as compared to the degree of substitutability of goods ($\lambda > \frac{b}{2}$), an improvement in terms of trade cost always encourages the government to subsidise more firms' R&D investment. By contrast, when the intensity of R&D spillovers is not so large relatively to the degree of substitutability of goods, whether trade liberalisation

[14] Once firms are allowed to make their domestic sale, the R&D cost function matters as it affects the price perceived by the consumer (and, hence, the consumer surplus).

increases or decreases the subsidy rate depends on the curvature of the R&D cost function (which capture how costly the activity is). As we know, when trade liberalisation occurs, firms enjoy more profits even if R&D spending is held fixed. If the R&D cost function is highly convex (i.e. R&D investment is a very costly activity), holding R&D investments fixed or even having a slight decrease in R&D efforts will allow firms to save a great deal of R&D spending. In terms of welfare, the society will be better off if firms do not change or even conduct less R&D because the savings (of R&D spending and R&D subsidy) obtained from doing so sufficiently offset any reduction in firms' profits and/or consumer surplus. To discourage firms from doing any further R&D, the government reduces its R&D subsidy extended to firms. However, when the R&D cost function is not so convex, because the marginal benefit from implementing an R&D project is greater than its corresponding cost, the government should encourage firms to do more R&D by increasing the R&D subsidy level in the face of trade liberalisation.

As for the impacts of trade liberalisation on the home economy, we can show that:

Proposition 4 *When firms compete in both home and foreign markets, at the optimal R&D subsidy, trade liberalisation in the foreign market: (i) increases a firm's R&D spending; (ii) increases the firm's export volumes, its domestic sales and, hence, its total sales; (iii) improves the firm's and industry productivity; and (iv) raises social welfare.*

Proof. See Appendix.

The results that trade liberalisation in the overseas market induces higher R&D spending of firms and, hence, lead to the improvement of their productivity as well as the industry productivity are, in general, similar to the case of no domestic sales investigated under Proposition 2. Trade liberalisation in the export market is not only welcomed by exporting firms as they can expand their output but also by their host country. This is because it makes the domestic economy as a whole become more efficient and, as a result, reap more welfare.

4 A foreign firm

So far, we have considered R&D policy when there is competition between exporting firms of which firms are domestic ones from the home government's perspective. This framework can easily be extended to encompass a foreign firm among them. In this new setting of one home firm competing with one foreign firm in both home and foreign markets, the home firm's response function would stay the same as in Section III. The home government's welfare slightly changes as it now includes profit of only one firm (i.e. the home firm), and the consumer surplus. Hence, technically, public R&D policy should qualitatively be similar to what was found in the previous section. To make things more interesting and possibly convey more policy implications, in what follows, we additionally assume different forms of R&D cooperation at firms' and governments' levels.[15] It should be noted that we continue to assume the exogenously given trade cost τ. We will not consider revenue generating tariffs because, as argued by Haaland and Kind (2008), these are not important for designing trade policy, especially in industrialised countries. In particular, we consider the following cases:

Case 1: R&D joint venture of firms
Suppose that the home firm and the foreign firm agree to form an R&D joint venture. As part of this agreement, the firms share research results with each other so that there is a maximum level of R&D spillovers or $\lambda = 1$. However, the firms still compete with each other in the output markets to maximise their own profits. The governments coordinate their R&D policies by setting a common level $s_h = s_f \equiv s$ in an effort to maximise total welfare $W = W_h + W_f$. Assume that firm i is the home firm and firm j is the foreign firm and denote $i = h$ and $j = f$ for the convenience of notation. The welfare function of the home government is $W_h = \frac{3q_{hh}^2}{2} + q_{hf}^2 + \frac{q_{fh}^2}{2} - r(x_h) + bq_{hh}q_{fh}$ and similarly for the foreign government's welfare function. Here, each national welfare is the sum of firm's profit and consumer surplus taking away any R&D subsidy cost (recall that we do not consider tariff revenue here).

Case 2: R&D policy cooperation
By R&D policy cooperation, we mean to consider the case of an asymmetric equilibrium in which the governments decide to entirely shut down one firm. Assume that the foreign firm is the one that is chosen to be shut down. The home government will be the one that subsidises R&D investment of the home firm in order to maximise the aggregate welfare. Welfare of the home government that includes the home firm's profit and the consumer surplus less the subsidy cost is $W_h = \frac{3q_{hh}^2}{2} + q_{hf}^2 - r(x_h)$. Meanwhile, welfare of the foreign government (including tariff revenue and consumer surplus) is $W_f = \frac{q_{hf}^2}{2}$. Hence, the global aggregate welfare is $W = W_h + W_f = \frac{3q_{hh}^2}{2} + \frac{3q_{hf}^2}{2} - r(x_h)$.

Upon deriving solutions to the welfare maximisation problem for each of the cases, we compare their welfare outcomes. We obtain the following results:

Proposition 5 *In an international setting with firms coming from different countries, while an international R&D joint venture of firms likely leads to a symmetric equilibrium, an international R&D cooperation at governmental level may trigger an asymmetric outcome in which only one firm is active in both R&D and production and receives financial support from its national government.*

[15]Previously, Suzumura (1992), Salant and Shaffer (1998, 1999) also consider R&D cooperation, however, between domestic firms. As a result, we will not consider this here but focus on the potential cooperation between a domestic firm and a foreign firm instead.

Proof. See Appendix.

Basically, the results say that when there is competition between a domestic exporting firm and a foreign firm, the home government may consider different policies of cooperation. The first policy option is to set up an R&D join venture and provide the home firm with an R&D subsidy when the fixed cost of R&D investment is not too high. This leads to a symmetric outcome at which the foreign government also follows suit. However, at a higher level of cooperation at policy level, an asymmetric outcome may arise. In particular, when the fixed cost of R&D investment is high enough, global aggregate welfare will be improved if there is only one firm conducting R&D investment.

5 Conclusion

In this paper we have considered different scenarios of competition between exporting firms to explore the effect of trade liberalisation in the foreign market and R&D policy on firms' incentive to innovate and social welfare. In particular, we studied in details the international setting in which exporting firms from the same country invest in R&D and sell their differentiated products in a foreign market. Here, R&D investment contains a positive externality. The home government uses R&D subsidy as a policy tool to maximise the social welfare. We showed that the magnitude of the R&D externality is an important factor, alongside the degree of substitutability between goods, that shapes the government's optimal policy behaviour. In particular, under some certain conditions involving these two factors, it might be optimal for the government to tax R&D instead of subsidising it. With this R&D tax put in place, trade liberalisation in the foreign market induces the home government to tax R&D more heavily as this policy response improves the domestic welfare. An asymmetric policy may arise in which one firm is heavily taxed on its R&D activity while the other receives no subsidy from the government. However, such a policy is not socially optimal. In addition, it requires a discriminatory policy from the government which is often restricted by laws.

In the next step, we examined if there are any changes in the results when firms also sell their products in the home market. It is found that the optimal policy for the home government in this case is to always provide financial support to firms' R&D activity (positive R&D subsidy). The impact of trade liberalisation on this optimal subsidy depends on the comparison between the R&D spillover effect and the degree of substitutability between goods and, to some extent, on the convexity of the R&D cost function.

We also extended our framework to consider the case of international competition between firms coming from different countries. We found that while an international R&D joint venture of firms likely leads to a symmetric equilibrium, an international

R&D policy cooperation at governmental level may trigger an asymmetric outcome. At this optimality, only one firm is active in both R&D and production and receives financial support from its national government. This arrangement is shown to be able to maximise aggregate welfare of the whole society.

Although the settings explored change from foreign market to both home and foreign markets, all in all, we found that trade liberalisation in the overseas market is always welfare enhancing as it induces higher output sales, both at home and overseas, of firms. It also entails a higher level of cost-reducing R&D spending which then leads to an improvement of firms' and industry productivity.

Overall, the results of our model are broadly in line with the literature stressing the complementarity between innovation and export: firms are more likely to export if they innovate and are more likely to innovate if they find good export opportunities (e.g. Lileeva and Trefler, 2010; Bustos, 2011). They are also well connected with previous works evaluating the aggregate effects of trade liberalisation (e.g. Eaton and Kortum, 2002; Bernard et al., 2003; Alvarez and Lucas, 2007; Alessandira and Choi, 2014). Although the attention in this paper is restricted to the competition of only two firms, the model can easily be extended to a multiple firm setting. Another direction is to ask the question about the extent to which empirical evidence confirms the theoretical predictions obtained in this paper. All these suggest an exciting future research agenda.

Appendix

Proof of Lemma 1

Conditional on the government's decision made regarding R&D subsidies in the first stage, each firm chooses how much to invest in R&D and how much to export in the second stage to maximise its profit defined in (3). Observe that$\{x_i, x_j, q_i, q_j\}$ must satisfy

$$x_i \geq 0, q_i \geq 0, x_i + \lambda x_j \leq c, q_i + bq_j \leq \alpha,$$
$$x_j \geq 0, q_j \geq 0, x_j + \lambda x_i \leq c, q_j + bq_i \leq \alpha,$$

Hence $\{x_i, x_j, q_i, q_j\}$ belongs to a compact set. The maximisation problem has a solution which is symmetric and can be written as

$$x_i = X(s_i, s_j), x_j = X(s_i, s_j), q_i = Q(s_i, s_j), q_j = Q(s_j, s_i).$$

The total welfare can be written as $W = \Pi_i(s_i, s_j) + \Pi_j(s_j, s_i)$. The problem $\max\{W : (s_j, s_j)\}$ will yield a symmetric solution $s_i = s_j$. When the solution is interior, the first order necessary conditions for firm i's profit maximisation problem give:

$$q_i + s_i - 2Mx_i = 0, \tag{11}$$

$$(\alpha - c - \tau) + x_i + \lambda x_j - bq_j - 2q_i = 0, \tag{12}$$

and similar for firm j.

In the first stage, the government, having known the firms' strategic response functions in (12) and (11), chooses R&D subsidy rates (s_i, s_j) to grant to firms in order to maximise the social welfare defined in (4) which can now be rewritten as $W = q_i^2 - r(x_i) + q_j^2 - r(x_j)$. Setting $\frac{\partial W}{\partial s_i} = 0$ and $\frac{\partial W}{\partial s_j} = 0$ yields the following:

$$2q_i.\frac{\partial q_i}{\partial s_i} - 2Mx_i.\frac{\partial x_i}{\partial s_i} + 2q_j.\frac{\partial q_j}{\partial s_i} - 2Mx_j.\frac{\partial x_j}{\partial s_i} = 0,$$

$$2q_i.\frac{\partial q_i}{\partial s_j} - 2Mx_i.\frac{\partial x_i}{\partial s_j} + 2q_j.\frac{\partial q_j}{\partial s_j} - 2Mx_j.\frac{\partial x_j}{\partial s_j} = 0,$$

where q_i and x_i (and, similarly, q_j and x_j) are given in (12) and (11). It can be seen that the first order conditions yield a symmetric outcome at which $s_i = s_j = s$, $q_i = q_j = q$, and $x_i = x_j = x$. Using this result to recalculate the social welfare we obtain $W = 2\left[\left(\frac{\alpha - c - \tau + (\lambda+1)x}{b+2}\right)^2 - Mx^2 - f\right]$. In what follows, we assume interior solutions for both profit maximisation of firms and welfare maximisation of the government. After differentiating this welfare function with respect to s, setting it to zero and using (11) and (12), we obtain $s = \frac{(2\lambda - b)q}{b+2}$.

Proof of Lemma 2

We divide the proof of this lemma into two parts. In the first part, we show the existence of an asymmetric policy. In the second part, we prove that such an asymmetric policy is not optimal from social welfare maximisation point of view.

The welfare function is given by $W = (\pi_i - s_i x_i) + (\pi_j - s_j x_j)$. Applying the Envelope Theorem[16] to the profit functions we have $\frac{\partial(\pi_i^* - s_i x_i^*)}{\partial s_i} = -s_i.\frac{\partial x_i^*}{\partial s_i}$ and $\frac{\partial(\pi_j^* - s_j x_j^*)}{\partial s_i} = -s_j.\frac{\partial x_j^*}{\partial s_i}$. This means that $\frac{\partial W}{\partial s_i} = -s_i.\frac{\partial x_i^*}{\partial s_i} - s_j.\frac{\partial x_j^*}{\partial s_i} = 0$ and $\frac{\partial W}{\partial s_j} = -s_i.\frac{\partial x_i^*}{\partial s_j} - s_j.\frac{\partial x_j^*}{\partial s_j} = 0$. Suppose that the government only wants firm i to conduct R&D so that $s_i \geq 0$, $x_i^* > 0$ and $s_j \leq 0$, $x_j^* = 0$. Given that $\frac{\partial x_i^*}{\partial s_i} > 0$ and $\frac{\partial x_i^*}{\partial s_j} < 0$, the above conditions require $s_i = 0$.

Now we turn to the first order conditions for firms' profit maximisation given in (12) and (11). Given that $s_i^A = 0$ and $x_j^A = 0$,[17] continuous substitution using these equations gives:

$$x_i^A = \frac{(\alpha - c - \tau)(2 - b)}{2M(4 - b^2) - (2 - b\lambda)},$$

[16] According to Milgrom and Segal (2002), in an optimisation problem with arbitrary choice sets, at any differentiability point of the value function, the Envelope Theorem holds if the objective function is differentiable in the parameter.

[17] The superscript A denotes the asymmetric case.

$$q_i^A = \frac{2M(\alpha - c - \tau)(2-b)}{2M(4-b^2)-(2-b\lambda)},$$

$$q_j^A = \frac{(\alpha - c - \tau)[2M(2-b)-(1-\lambda)]}{2M(4-b^2)-(2-b\lambda)},$$

$$s_j^A = -\frac{(\alpha - c - \tau)[2M(2-b)-(1-\lambda)]}{2M(4-b^2)-(2-b\lambda)},$$

$$W^A = \left[(2M(2-b)-(1-\lambda))^2 + 4M^2(2-b)^2 - M\right]\left[\frac{\alpha - c - \tau}{2M(4-b^2)-(2-b\lambda)}\right]^2.$$

One can verify that under Assumption 1, $0 < x_i^A < c$, $q_i^A > 0$, $q_j^A > 0$ and $s_j^A < 0$. In addition, the conditions that $q_i^A + bq_j^A < \alpha$ and $q_j^A + bq_i^A < \alpha$ are also met. This means that an asymmetric policy may arise from the current setting.

To show that this asymmetric policy arrangement is not socially optimal, we compare its welfare with that obtained under the symmetric equilibrium setting. The symmetric equilibrium yields:[18]

$$x^S = \frac{(\alpha - c - \tau)(\lambda + 1)}{M(b+2)^2 - (\lambda+1)^2},$$

$$q^S = \frac{(\alpha - c - \tau)M(b+2)}{M(b+2)^2 - (\lambda+1)^2},$$

$$W^S = \frac{2M(\alpha - c - \tau)^2}{M(b+2)^2 - (\lambda+1)^2}.$$

Again, it can be verified that under Assumption 1, $0 < x^S < c$ and $0 < (1+b)q^S < \alpha$. The issue of comparing W^A with W^S simplifies to comparing Ω_1 with Ω_2 where these denote the following:

$$\Omega_1 = 2M.[2M(4-b^2)-(2-b\lambda)]^2,$$

$$\Omega_2 = [(2M(2-b)-(1-\lambda))^2 + 4M^2(2-b)^2 - M].[M(b+2)^2 - (\lambda+1)^2].$$

Upon carefully computing $\Omega_1 - \Omega_2$ we reach the result that $\Omega_1 - \Omega_2 > 0$. This implies that $W^S > W^A$. In other words, the asymmetric policy does not yield an optimal outcome from the social welfare point of view.

Proof of Proposition 1

We will prove this proposition in two parts. In the first part, we prove the existence of a unique value of s. We then indicate that s can either be positive (i.e. an optimal subsidy) or negative (i.e. an optimal tax) depending on values of relevant parameters. In the last part, we examine the comparative statics on this policy variable with regard to a decrease in τ (trade liberalisation).

[18] The superscript S indicates the symmetric case.

When $\lambda \neq \frac{b}{2}$, for any given level of subsidy provided from the government, the equilibrium export volume is:

$$q = \frac{(b+2)s}{2\lambda - b}. \tag{13}$$

Inserting the result in (13) into (12) and (11) under symmetry delivers:

$$x = \frac{(b+2)^2 s}{(2\lambda - b)(\lambda + 1)} - \frac{(\alpha - c - \tau)}{\lambda + 1}. \tag{14}$$

Because export volume and R&D investment are non-negative, we must have $\frac{s}{2\lambda - b} > 0$. This implies either $s > 0$ when $\lambda > \frac{b}{2}$ or $s < 0$ when $\lambda < \frac{b}{2}$.

To simplify notation, let $\theta = \frac{s}{2\lambda - b} > 0$. We next identify conditions that need to be imposed on θ to make sure that the firms' profit maximisation problem yield interior solutions. More specifically, we need $0 < (1+b)q < \alpha$ and $0 < (\lambda+1)x < c$. While the first condition guarantees positive quantities and prices of the goods, the second one is necessary for having plausible positive R&D investments. Using the result in (13), the double inequalities $0 < (1+b)q < \alpha$ imply $0 < \theta < \frac{\alpha}{(b+1)(b+2)}$. Using (14), the double inequalities $0 < (\lambda+1)x < c$ imply $\frac{(\alpha - \tau)}{(b+2)^2} > \theta > \frac{(\alpha - c - \tau)}{(b+2)^2}$. Combining these two results, the range of value for θ is $\frac{(\alpha - \tau)}{(b+2)^2} > \theta > \frac{(\alpha - c - \tau)}{(b+2)^2}$.

It can be seen that the function W is strictly concave in x. Indeed, we have $\frac{\partial^2 W}{\partial x^2} = 2\left[\frac{2(\lambda+1)^2}{(b+2)^2} - 2M\right]$. Since $2M(b+2)^2 > (b+5)(\lambda+1)^2 > 2(\lambda+1)^2$, $\forall b \in (0,1)$, $\forall \lambda \in [0,1]$ as per Assumption 1 then $\frac{\partial^2 W}{\partial x^2} < 0$. Because x is affine in s according to (14), W is also strictly concave in s. We will next show that there exists a unique interior solution to the government's welfare maximising problem.

Substituting the obtained results into (11) gives:

$$(\lambda + 1)\theta - Mx = 0. \tag{15}$$

This, together with (14), yields:

$$s = \frac{M(2\lambda - b)(\alpha - c - \tau)}{M(b+2)^2 - (\lambda+1)^2}. \tag{16}$$

It can be verified that under Assumption 1, this result satisfied the condition set out for the range of value of θ. This is the unique optimal R&D policy measure that should be applied by the government to the firms' R&D efforts in order to maximise the social welfare. When $\lambda > \frac{b}{2}$, $s > 0$, there is an optimal R&D subsidy conducted. However, when $\lambda < \frac{b}{2}$, $s < 0$, it is optimal to have an R&D tax instead.

Regarding the impact of trade liberalisation in the overseas market, differentiating (16) with respect to τ and rearranging we get $\frac{\partial s}{\partial \tau} = -\frac{M(2\lambda - b)}{M(b+2)^2 - (\lambda+1)^2}$. Clearly, if $\lambda > \frac{b}{2}$, $s > 0$, and $\frac{\partial s}{\partial \tau} < 0$. As this is the case of an optimal R&D subsidy, other things equal, trade liberalisation (a smaller τ) induces a higher level of optimal R&D subsidy provided to firms. By contrast, if $\lambda < \frac{b}{2}$, $s < 0$, and $\frac{\partial s}{\partial \tau} > 0$. In this case, a decrease in τ leads to a corresponding decrease in s (s becomes more negative). In other words, a higher level of optimal R&D tax should be levied.

Proof of Proposition 2

The proof of this proposition is quite straightforward. Indeed, making use of (14) and (17), we get $\frac{\partial x}{\partial \tau} = -\frac{M(b+2)^2}{(\lambda+1)[M(b+2)^2-(\lambda+1)^2]} < 0$ and $\frac{\partial q}{\partial \tau} = -\frac{M(b+2)}{M(b+2)^2-(\lambda+1)^2} < 0$. These mean that trade liberalisation (lower τ) leads to an expansion of both R&D investments and export volumes of firms at the optimal policy measure that the government conducts.

Due to symmetry, in equilibrium, firms' and industry productivity are the same $Z = z = \frac{1}{c-(\lambda+1)x}$. Differentiating this with respect to τ delivers $\frac{\partial Z}{\partial \tau} = \frac{\partial z}{\partial \tau} = \frac{\lambda+1}{[c-(\lambda+1)x]^2} . \frac{\partial x}{\partial \tau} < 0$. A decrease in the trade cost helps strengthen firm as well as the industry average productivity. Regarding what happens to the whole society, the effect on welfare is:

$$\frac{\partial W}{\partial \tau} = 4\left[\frac{\partial s}{\partial \tau} . \frac{(b+2)^2}{2\lambda - b}\left(\theta - \frac{Mx}{\lambda+1}\right) - \frac{Mx}{\lambda+1}\right].$$

A close look at the first term inside the square bracket indicates that it is equal to zero according to equation (15). Hence, $\frac{\partial W}{\partial \tau} < 0$ or W is decreasing in τ. A fall in τ will increase W or welfare increases with trade liberalisation in the foreign market.

Proof of Lemma 3

The first order conditions from firm i's profit maximisation problem are:

$$q_{ih} + q_{if} + s_i - 2Mx_i = 0, \tag{17}$$

$$(\alpha - c) + x_i + \lambda x_j - bq_{jh} - 2q_{ih} = 0, \tag{18}$$

$$(\alpha - c - \tau) + x_i + \lambda x_j - bq_{jf} - 2q_{if} = 0, \tag{19}$$

and similar for firm j.

In the first stage, the aggregate welfare takes the following form:

$$W_h = \frac{3q_{ih}^2}{2} + q_{if}^2 - r(x_i) + \frac{3q_{jh}^2}{2} + q_{jf}^2 - r(x_j) + bq_{ih}q_{jh}.$$

The government's welfare maximisation delivers the first order conditions:

$$3q_{ih}.\frac{\partial q_{ih}}{\partial s_i} + 2q_{if}.\frac{\partial q_{if}}{\partial s_i} - r'(x_i).\frac{\partial x_i}{\partial s_i} + 3q_{jh}.\frac{\partial q_{jh}}{\partial s_i} + 2q_{jf}.\frac{\partial q_{jf}}{\partial s_i} - r'(x_j).\frac{\partial x_j}{\partial s_i} + bq_{ih}.\frac{\partial q_{jh}}{\partial s_i} + bq_{jh}.\frac{\partial q_{ih}}{\partial s_i} = 0,$$

$$3q_{ih}.\frac{\partial q_{ih}}{\partial s_j} + 2q_{if}.\frac{\partial q_{if}}{\partial s_j} - r'(x_i).\frac{\partial x_i}{\partial s_j} + 3q_{jh}.\frac{\partial q_{jh}}{\partial s_j} + 2q_{jf}.\frac{\partial q_{jf}}{\partial s_j} - r'(x_j).\frac{\partial x_j}{\partial s_j} + bq_{ih}.\frac{\partial q_{j1}}{\partial s_j} + bq_{jh}.\frac{\partial q_{i1}}{\partial s_j} = 0,$$

where q_{ih}, q_{if}, and x_i (and similar for q_{jh}, q_{jf}, and x_j) are given in (17) - (19). These equations imply a symmetric outcome where $s_i = s_j = s$, $q_{ih} = q_{jh} = q_h$, $q_{if} = q_{jf} = q_f$, and $x_i = x_j = x$. Using this symmetric result to recalculate the social welfare we get $W_h = (b+3)q_h^2 + 2q_f^2 - 2r(x)$, which in turn implies the following after re-deriving the first order condition: $q_h\left[(3+b)\lambda+1\right] + q_f(2\lambda-b) - (b+2)s = 0$. Using this result, we can figure out:

$$q_h = \frac{(b+2)s}{(b+5)\lambda+1-b} + \frac{(2\lambda-b)\tau}{(b+2)\left[(b+5)\lambda+1-b\right]},$$

$$q_f = \frac{(b+2)s}{(b+5)\lambda+1-b} - \frac{\left[(b+3)\lambda+1\right]\tau}{(b+2)\left[(b+5)\lambda+1-b\right]},$$

$$x = \frac{(b+2)^2 s}{(\lambda+1)\left[(b+5)\lambda+1-b\right]} + \frac{(2\lambda-b)\tau}{(\lambda+1)\left[(b+5)\lambda+1-b\right]} - \frac{\alpha-c}{\lambda+1}.$$

Now, we check for the second order condition:

$$\frac{\partial^2 W}{\partial s^2} = \frac{2(b+2)^2}{\left[(b+5)\lambda+1-b\right]^2}\left[b+5-2M.\frac{(b+2)^2}{(\lambda+1)^2}\right] < 0.$$

Hence, the second order condition is satisfied for a maximum.

To make sure that quantities and prices are positive, we need to impose that $0 < (\lambda+1)x < c$, and $0 < (b+1)q_h < \alpha$, as well as $0 < (b+1)q_f < \alpha$. These lead to the following conditions:

$$\frac{\left[(b+5)\lambda+1-b\right](\alpha-c) - (2\lambda-b)\tau}{(b+2)^2} < s < \frac{\left[(b+5)\lambda+1-b\right]\alpha - (2\lambda-b)\tau}{(b+2)^2}. \tag{20}$$

Clearly, under the assumption on the trade cost, the lower bound is positive. This means that s is always positive.

Proof of Proposition 3

Substituting the results for q_h and q_f given in the proof of Lemma 3 into (17) and rearranging gives:

$$\frac{(b+5)(\lambda+1)s}{(b+5)\lambda+1-b} - \frac{(b+1)(\lambda+1)\tau}{(b+2)\left[(b+5)\lambda+1-b\right]} - 2Mx = 0. \tag{21}$$

This allows us to derive:

$$x = \frac{(b+5)(\lambda+1)s}{2M\left[(b+5)\lambda+1-b\right]} - \frac{(b+1)(\lambda+1)\tau}{2M(b+2)\left[(b+5)\lambda+1-b\right]}.$$

It can be seen that x is increasing in s. Given the range of s specified in the proof of Lemma 3 then the range of value of x should be $\frac{(\lambda+1)[(b+5)\alpha-2\tau]}{2M(b+2)^2} > x > \frac{(\lambda+1)[(b+5)(\alpha-c)-2\tau]}{2M(b+2)^2}$.

Obviously, $\frac{(\lambda+1)[(b+5)(\alpha-c)-2\tau]}{(b+2)^2} > 0$ because $\alpha - c - \tau > 0$. Hence, as soon as $\frac{(\lambda+1)[(b+5)\alpha-2\tau]}{2M(b+2)^2} < c$ or $\frac{(\lambda+1)[(b+5)\alpha-2\tau]}{2c(b+2)^2} < M$ (which is satisfied as per Assumption 1) then x solves (21). This, in turn, allows us to get:

$$s = \frac{(b+1)(\lambda+1)^2 + 2M(2\lambda-b)(b+2)}{(b+2)\left[(b+5)(\lambda+1)^2 - 2M(b+2)^2\right]}.\tau - \frac{2M(\alpha-c)\left[(b+5)\lambda+1-b\right]}{\left[(b+5)(\lambda+1)^2 - 2M(b+2)^2\right]}.$$

This means that s is positive and unique. Differentiating the obtained result with respect to τ gives:

$$\frac{\partial s}{\partial \tau} = \frac{(b+1)(\lambda+1)^2 + 2M(2\lambda-b)(b+2)}{(b+2)\left[(b+5)(\lambda+1)^2 - 2M(b+2)^2\right]}. \tag{22}$$

It should be noted that the denominator of this partial derivative is always negative. As for the numerator, it is positive if $\lambda \geq \frac{b}{2}$ or $\lambda < \frac{b}{2}$ and $\frac{(b+5)(\lambda+1)^2}{2(b+2)^2} < M < \frac{(b+1)(\lambda+1)^2}{2(b-2\lambda)(b+2)}$. In that case the whole fraction $\frac{\partial s}{\partial \tau} < 0$ or s is decreasing in τ. A decrease in τ will result in an increase in s at the optimal. When $M > \frac{(b+1)(\lambda+1)^2}{2(b-2\lambda)(b+2)}$ for $\lambda < \frac{b}{2}$ the numerator is negative so $\frac{\partial s}{\partial \tau} > 0$ or s is increasing in τ. When $M = \frac{(b+1)(\lambda+1)^2}{2(b-2\lambda)(b+2)}$ for $\lambda < \frac{b}{2}$, $\frac{\partial s}{\partial \tau} = 0$ implying that s is unaffected by a change in τ.

Proof of Proposition 4

Using the results for q_h, q_f and x in the proof of Lemma 3 and then (22), we obtain the following partial derivatives:

$$\frac{\partial q_h}{\partial \tau} = \frac{2(\lambda+1)^2}{(b+2)\left[(b+5)(\lambda+1)^2 - 2M(b+2)^2\right]} < 0,$$

$$\frac{\partial q_f}{\partial \tau} = \frac{2M(b+2)^2 - (b+3)(\lambda+1)^2}{(b+2)\left[(b+5)(\lambda+1)^2 - 2M(b+2)^2\right]} < 0,$$

$$\frac{\partial x}{\partial \tau} = \frac{2(\lambda+1)}{\left[(b+5)(\lambda+1)^2 - 2M(b+2)^2\right]} < 0.$$

Defining $q = q_h + q_f$ as a firm's total sales then $\frac{\partial q}{\partial \tau} = \frac{\partial q_h}{\partial \tau} + \frac{\partial q_f}{\partial \tau} < 0$. The industry productivity is equal to firm's productivity $Z = z = \frac{1}{c-(\lambda+1)x}$. Differentiating this with respect to τ delivers $\frac{\partial Z}{\partial \tau} = \frac{\partial z}{\partial \tau} = \frac{\lambda+1}{[c-(\lambda+1)x]^2}.\frac{\partial x}{\partial \tau} < 0$. As for the welfare effect, we have:

$$\frac{\partial W}{\partial \tau} = (b+3).2q_h.\frac{\partial q_1}{\partial \tau} + 4q_f.\frac{\partial q_2}{\partial \tau} - 4Mx.\frac{\partial x}{\partial \tau}.$$

Substituting (21) and the results derived above into this equation and simplifying we get:

$$\frac{\partial W}{\partial \tau} = \frac{4\left\{\left[(b+3)\lambda+1\right]\tau - s(b+2)^2\right\}}{(b+2)^2\left[(b+5)\lambda+1-b\right]}.$$

Note that the denominator of this fraction is positive. Given the range of value of s in

(20), we can work out the following:

$-\left[(b+5)\lambda+1-b\right](\alpha-\tau) \leq \left[(b+3)\lambda+1\right]\tau - s(b+2)^2 \leq -\left[(b+5)\lambda+1-b\right](\alpha-\tau-c)$.

This means $\left[(b+3)\lambda+1\right]\tau - s(b+2)^2 < 0$. Hence, we can conclude that $\frac{\partial W}{\partial \tau} < 0$.

Proof of Proposition 5

From the setting for the R&D joint venture (JV) of firms, after solving the home firm's profit maximisation and the aggregate welfare maximisation for both governments, we obtain the following interior solutions under symmetry (i.e. $q_{hh} = q_{ff} = q_1^{JV}$, $q_{fh} = q_{hf} = q_2^{JV}$, $x_h = x_f = x^{JV}$ and $s_h = s_f = s^{JV}$):

$$q_1^{JV} = \frac{(b+2)s}{2(b+4)} + \frac{\tau}{2(2-b)},$$

$$q_2^{JV} = \frac{(b+2)s}{2(b+4)} - \frac{\tau}{2(2-b)},$$

$$x^{JV} = \frac{(b+2)^2 s}{4(b+4)} + \frac{\tau - 2(\alpha-c)}{4},$$

where $\frac{2(\alpha-c)(b+4)}{(b+2)^2} < s < \frac{(2\alpha-\tau)(b+4)}{(b+2)^2}$ is required for the interior solutions to be established. It can be verified that these obtained results also fulfil the second order sufficient conditions for the maximisation problems.

From the setting for R&D policy cooperation (CO), upon denoting $q_{hh} = q_1$, $q_{hf} = q_2$ and $x_h = x$ and resolving the maximisation problems, we obtain:

$$q_1^{CO} = s + \frac{\tau}{4},$$

$$q_2^{CO} = s - \frac{\tau}{4},$$

$$x^{CO} = 2s + \frac{\tau - 2(\alpha-c)}{2},$$

where the condition on s is $\frac{2(\alpha-c)-\tau}{4} < s < \frac{2\alpha-\tau}{4}$.

We can also figure out the following for the subsidy rates under alternative arrangements:

$$s^{JV} = \frac{M(b+4)\left[2(\alpha-c)-\tau\right]}{M(b+2)^2 - 4(b+3)},$$

$$s^{CO} = \frac{M\left[2(\alpha-c)-\tau\right]}{4M-3}.$$

It can be shown that $s^{CO} > s^{JV}$ meaning that the subsidy required to totally shut down one firm is more than the case when firms are allowed to form an R&D joint venture.

Based on these results, we can calculate the welfare for each scenario as follows:

$$W^{JV} = \frac{(3-b)\tau^2}{(2-b)^2} + \frac{M(b+3)\left[2(\alpha-c)-\tau\right]^2}{2\left[M(b+2)^2-4(b+3)\right]} - 2f,$$

$$W^{CO} = \frac{3\tau^2}{16} + \frac{3M\left[2(\alpha-c)-\tau\right]^2}{4(4M-3)} - f.$$

Therefore, we have:

$$W^{CO} - W^{JV} = \frac{\left(3b^2+4b-36\right)\tau^2}{16(2-b)^2} + \frac{M\left[2(\alpha-c)-\tau\right]^2\left[3M(b+2)^2-2(4M+3)(b+3)\right]}{4(4M-3)\left[M(b+2)^2-4(b+3)\right]} + f.$$

Clearly, for $b \in [0,1)$ then $\frac{\left(3b^2+4b-36\right)\tau^2}{16(2-b)^2} < 0$. Now we examine the second term of the above equation keeping in mind that the denominator is always positive. For the numerator, because $M\left[2(\alpha-c)-\tau\right]^2 > 0$, we take a closer look at things inside the long square bracket. We have:

$$3M(b+2)^2 - 2(4M+3)(b+3) < M\left[3(b+2)^2 - 8(b+3)\right] < 0.$$

While the first inequality is obvious, the second one is a direct result of the assumption on the range of b (i.e. $b \in [0,1)$). This means that the second term of the welfare difference is negative as well. In the extreme case where $f = 0$, we will get $W^{CO} < W^{JV}$. As the R&D fixed cost gets larger, the incentive for R&D policy cooperation will also get larger. Once this fixed cost is above the threshold f_1 then the whole society will be better off to run an asymmetric equilibrium by completely shutting down one firm and subsidising the other. The value of the threshold is:

$$f_1 = \frac{\left(36-3b^2-4b\right)\tau^2}{16(2-b)^2} + \frac{M\left[2(\alpha-c)-\tau\right]^2\left[2(4M+3)(b+3)-3M(b+2)^2\right]}{4(4M-3)\left[M(b+2)^2-4(b+3)\right]}.$$

We can also calculate the following:

$$W_h^{CO} - W_f^{CO} = \frac{\tau^2}{8} + \frac{M(8M-9)\left[2(\alpha-c)-\tau\right]^2}{4(4M-3)^2} + \frac{M\tau\left[2(\alpha-c)-\tau\right]}{2(4M-3)} - f.$$

This indicates that $W_h^{CO} > W_f^{CO}$ if $f < f_2$ where

$$f_2 = \frac{\tau^2}{8} + \frac{M(8M-9)\left[2(\alpha-c)-\tau\right]^2}{4(4M-3)^2} + \frac{M\tau\left[2(\alpha-c)-\tau\right]}{2(4M-3)}.$$

It can be seen that when $f_1 < f \leq f_2$, the optimal outcome is an asymmetric equilibrium in which the home firm conducts R&D investment and receives R&D subsidy from its government and the home country enjoys a little higher welfare. However, when $f > f_2$, the home country does not have any incentive to conduct R&D as it obtains a lower welfare level than that of the foreign counterpart.

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